Infinis

I. M. R. Pinheiro

Abstract
The sigmatoid infinity has always had a troubled mathematical reference. Some think that such a sigmatoid exists for them to be 'saved from thinking for too long about a few special issues'. These may say, it will take me an infinity of time to do this (therefore I will not do it)! Others, like the researchers working with Calculi, see infinity as a logically correct tag for everything that, in principle, does never end: Adding to infinity, multiplying to infinity, and so on so forth. In this paper, we criticize the choice of the sigmatoid infinity to point at the reference in the minds of the researchers who work with Calculi nowadays, propose suitable replacement for it, and create a new system of geometric representation and reference for mathematical objects to make it possible for Mathematics to adapt to the proposed changes.

1. Introduction

The sigmatoid infinity apparently comes from the Latin sigmatoid 'infinis', which is found associated with the reference 'no borders' (see [S. Schwartzman 1994], for instance).

Since the term is seen in the writings of professional mathematicians in sentences such as 'there is an infinity of numbers between any randomly chosen couple of numbers in the universe of the real numbers' as well as in sentences such as 'the highest value in the reals is infinity', one would think that either the two randomly chosen numbers are not borders or infinity, in Mathematics, has at least two different references.

Because the two randomly chosen numbers definitely form borders for all those 'in-between', it can only be that infinity, in Mathematics, has at least two different references, at least one of those being, in nature, very different from the etymological one.

The problem with that is that every term of the mathematical lingo must point exclusively at the intended abstract reference and therefore words cannot be imported from the natural language and changed in their meaning 'to suit our intentions' in Mathematics, first of all. We also cannot have sigmatoids that allow for association with more than one mathematical reference inside of the mathematical lingo, for the main qualities of the mathematical lingo are clarity and objectivity (1).

To guarantee that the just-mentioned principles be respected ((1)), we demand a perfect bijection when adding a new term to the mathematical lingo (2).

It is obvious that the infinity the never-ending-progress of the horizontal Cartesian axis forms is of different nature -completely different- from that that the impossible-to-be-counted real numbers in the middle of other two forms: Not only the numbers in the middle will eventually end in a rational, but, were there lines, especially built for the visits to 'Mr. Infinity', they would fold at an impossible-to-be-identified place and come back to the first rational number after the departure rational number.

Truth is that if both mathematical references could be paired with the same pointer, such could only be in the English language, never in Mathematics, because, in Mathematics, we need to respect the pertinence condition (2).

How could something, which is reached in the middle of something else, compare to something that nothing ever reaches?

Infinity is a sigmatoid of language, and therefore it should not belong to the universe of Mathematics. Because of the condition (2), we can actually assert that including 'infinity' in the mathematical lingo was simply a mistake.

There were several evidences, readily available, throughout human history, on infinity not being an adequate choice of sigmatoid for the mathematical lingo, but they all have been fully ignored up to this paper, as for all we know now.

Amongst those, we have, for instance, the creation of Aleph. Aleph would be 'the last number', which would not be infinity, but would correspond to it (?) …

Mathematics should not be inspiring and mysterious, but objective, so that Aleph should belong at most to poetry, music, and alike 'universes' of the human existence.

Interesting details on Aleph are found in [H. C. Parr 2003], for instance. According to the source, Aleph-0 would correspond to the order of the infinity to which the natural numbers would belong, what does make sense: A trial of finding a suitable replacement for the Euclidean distance in this situation, something that would allow us to measure how much bigger one infinity is when compared to another. The same source, however, brings trivial argumentation as to why Aleph-0 is not a good measure for how big the infinity of the natural numbers is: They multiply each one of the natural numbers by two and obtain the set of the even numbers, and this set then is half of the size of the set of the natural numbers, but is still of Aleph-0 size.

The most valuable insight that the discussions on Aleph bring to us is that we should worry about the nature and the size of the infinities.

We then must remember that the first question to be asked is 'why?', not 'how?', in any serious theory.

In this paper, we not only manage to fix the existing concept of infinity, replacing its sigmatoid with something more properly included in Mathematics, but we also describe, at least partially, the universe in which the current reference for the concept of infinity inhabits - universe previously inaccessible to mathematicians - via the own Mathematics.

We start with a discussion on the Latin term 'infinis', progress to try to find a sigmatoid to point at the amount of real numbers between any two randomly chosen natural numbers, write about the World of Infinita (WI), exemplify the use of the WI, and finish the paper with both the conclusion and the reference sections.

2. Infinis

There is no more adequate translation than 'that which has got no boundaries' for what happens to the growth of the real numbers, as time goes by, in any step you take over them (one unit, half a unit, and etc.). Walking in the positive direction over the real numbers line, for instance, would lead the walker to the conclusion that the only possible answer to the question 'what is the highest real number?' is 'the real numbers have (infinis) no borders', that is, they go nowhere or they keep on going forever. Because mathematicians are lazy, instead of stating 'real numbers have 'infinis'', they say that they go to infinity, meaning that they go to 'the special place' where no borders exist, or to no place at all, since there is never a stopping point.

The entry infinity is found in ([Merriam-Webster 2009]), one of our most popular dictionaries, like this:

Main Entry:
in·fin·i·ty
Pronunckiation:
\in-'fi-nə-tē
Function:
noun
Inflected Form(s):
plural in·fin·i·ties
Date:
14th century

1 a: the quality of being infinite
b: unlimited extent of time, space, or quantity : boundlessness
2: an indefinitely great number or amount <an *Infinity* of stars>3 a: the limit of the value of a function or variable when it tends to become numerically larger than any preassigned finite number b: a part of a geometric magnitude that lies beyond any part whose distance from a given reference position is finite <do parallel lines ever meet if they extend to *Infinity*> c: a transfinite number (as Aleph-null)4: a distance so great that the rays of light from a point source at that distance may be regarded as parallel

There is substantial difference between a precise place called 'infinity' (meaning number 3a from the dictionary extract) and the quality of 'having no borders' (meaning 1b from the dictionary extract). If there is a place where x, for instance, dies, x must be of finite nature, but if it has got no borders, it keeps on going forever, never dying. Therefore, it is the hugest mistake of all stating that the real numbers 'go to infinity'. No, they 'keep on going forever', never stopping, at most 'with infinis'. Notwithstanding, if forcing the abstract reality and language into an impossible match, we must make it all clear in the Mathematics for, in Mathematics, neither mistakes nor inaccuracies are allowed (therefore nothing that may generate multiple interpretations, especially in terms of definitions).

The best way to go would then be excluding the sigmatoid from the mathematical lingo, as well as the symbol, and, as before the creation of 'infinity', the limit of something that does not have a limit would be told to be what it is: Impossible to calculate because it does not exist, therefore not something acceptable if we talk about mathematical operations.

Another reason to simply exclude, from the set of possible questions in Mathematics, 'what is the limit of the real function f(x) = x when x goes to infinity?', is the psycholinguistic subconscious implications regarding the 'existence allowance' for this question in Mathematics: Whatever the human brains cannot accept, it will simply repeat, in terms of educational tokens. Because of that, the existence allowance for this sort of question in Mathematics will generate not only confusion and conflict in the understanding of the notion for the literate user of the mathematical lingo (because of the gap that then appears between what they grasp from a concept and what they are obliged to accept as an imposition), but it will also make those support equivocated theorems and theories.

The limit of the function f(x) = x, when x grows to the last number in the reals (that means never stopping), has to be delusional, for the last number does not exist! We should state, in good 'mathematical terms', that is, in terms that, at least as a set, when together, form perfect bijection with their references, that 'the limit of f(x) = x, as x approaches the last number in the set of the real numbers, is not a valid mathematical request because there is no last number in the real axis' (writing things as they are is obviously the most basic mathematical duty of all).

In wondering about the need that human beings seem to have created with time (at the beginning, mathematicians used to simply state that such a limit did not exist. Such statement was not as good as we would like it to be and impacted negatively on the mathematical theories (not all negative impact has to do with psycholinguistics. Some cases of ratios involving numerator and denominator that contain infinity have standard solutions, for instance)), that 'of having a symbol' for an inexistent geometric place, we have come up with the idea of a 'special space' that could be used to 'host' this 'special entity' that we believe should belong to the mathematical universe but, for one reason or another, cannot be accepted as a genuine part of it.

The creation of this special space, beyond Mathematics, or simply outside of it, would both satisfy the 'almost scientifically' identified psychological need of mathematicians (regarding the symbol and the sigmatoid to designate the mental references 'place that we cannot access neither with our body nor with our imagination' and 'impossible-to-be-imagined amount') and finish with the psycholinguistic harmful implications regarding the psychologically needed entity.

When we created this space, we had in our minds that the most basic mistake, in terms of theory, that we could identify, in all that we have mentioned this far, was assuming that infinity is a geometric place: If something is nothing that we can 'touch' with our bodies or reach with our imagination, then it is definitely not a place …

To 'fix the concept', we had to first create a way of clearly stating that the sigmatoid 'infinity' did not belong to the world of Mathematics, but to somewhere connected, a somewhere of different nature to that of Mathematics. After reading [Bunyan 1997], we have decided to call such a somewhere *Extramathematics*.

As an illustrative allurement Extramathematics would be to Mathematics the same thing that Metaphysics is for human beings: A sort of connection between what is of their nature and what is beyond. This way, just like when humans 'transcend' their nature (human) they reach God, when numbers transcend theirs, they reach infinity.

In Extramathematics, just like they currently have in Mathematics, there are a few symbols to point to nothing in particular, to refer to the 'absence of something', to replace some linguistic expressions because we wish to write as little as possible.

In Mathematics, at this point in time, 'is not a sound mathematical request' is replaced with 'is equal to lazy eight' in a few operations involving limits, for instance.

In the just-mentioned substitution, of a univocal linguistic expression for a symbol, the main principles of Mathematics are currently being all grossly disrespected.

Basically, the own number, 'poor thing', will never get to infinity, in the same way that humans will never get to God: There is always a lot in the middle, not mattering the names (soul, Jesus, and etc.); it is as if God and Infinity did not actually exist, what existed were everything in the middle, but if we did not invent God and Infinity, we would lose the 'anthropocentric' (human beings apparently love to be commanded, want to adore things, and believe that they will be able to occupy the position of whatever to them be superior someday. If the figure of the boss did not exist, humans apparently would feel lost, in a world with no leader to follow, that is, with nobody to be blamed for their mistakes that be not themselves) nature of things, and we would be unable to understand all.

We have, a few lines ago, transferred a few mathematical entities and expressions to the *Extramathematics* world. Now, we need to transfer them to the Mathematics World once more. For that purpose, we will make use of our 'World of Infinitum', where nothing will ever reach, mainly because it does not exist, but we imagine it does, and we will work with the imaginary infinitum in the same way that we work with the imaginary axis, for the square root of a negative number also does not exist.

3. World of Infinita

Both mathematicians and logicians (this far in time) have never really acknowledged the enormous differences between Language and Mathematics, perhaps not even at a school level.

For instance, suppose that one states, in the English language, 'I will go to infinity!'. Is that right or wrong?

The answer is simple: There is no right or wrong there, for a person may utter whatever they like, sensible or not; there are no rules for utterances. There are fixed rules for punctuation, spelling, and quite a few other things, but not for what may, or may not, be uttered.

We say 'I will go to infinity!' and the message we wish to convey is sometimes even fully understood by the intended recipient.

It is probably something similar to 'I will go to where you cannot find me'.

Mathematics, aiming to be part of Science, must hold severely tied discourse, which allow for both progress to happen and abstraction to be built over abstraction. We should always worry about the study of its terms, so that no mistakes occur, especially inconsistencies.

In language, the idea of place, when using the term 'infinity', is unavoidable: 'Look at the infinity! (Means skies, usually)' is one of the most trivial examples.

We also say 'myriad, or infinity, of coins (means uncountably many in our heads but, for Mathematics, when this is used in language, it actually means countably many, with no exception, for the number of coins produced on Earth is always finite)'. The idea, in the case with the coins, is that we get exhausted of trying to count the coins. Infinity, in this case, appears to be associated with the size of the sky or to a 'special place': A place of fatigue, relating to the inability of determining things from a human perspective.

We seem to have extended the same 'privileges' of the human language to Mathematics, unfortunately, without considering that while in language anything, almost, is acceptable, in Mathematics, very little is passive of acceptance, with every term of its technical lingo needing to be, first of all, a univocal term.

We have wrongly called 'infinity' both the amount of numbers 'around a specific number in the real axis' and the 'place that is not determined, and is never reached by means of eyes or imagination'.

The two ideas are substantially different in nature and, therefore, could only increase mistake, and probability of mistake, in Science, if taken to be 'the same', that is, if being assigned the same sigmatoid to designate them in the mathematical lingo. Our concern, with the terms of the mathematical lingo, is always that of the 'perfect bijection', that is, that the term univocally point to a certain object (of either concrete or abstract nature) and that the object univocally point to the term in return.

One could, at this stage, easily state that we should simply drop the name and come up with another name, not common in language, created for that specific end. That is obviously the only way to go, since the term 'infinity' has already been coined (normal language) and will always, therefore, hold ambiguous meaning if added to the mathematical lingo.

Proceeding this way, we start with taking the smallest piece that we may physically get in a ruler to mean X. We then may say that X is a unit of measurement for the ruler. Next, we assume that each unit forms 'one infinitum'. Then the set of all 'infinita' will form the whole ruler, or axis. With this, instead of referring to infinity as a boundary, we will refer to the supremum of the infinita, or the infimum of the minus infinita, both concepts with no reference either in the ruler or in the world we live -empty sigmatoids- but, at least, the right idea in Mathematics, with no mistake. We may then accurately describe, in mathematical terms, what before was wrongly described, generating inaccuracies in the mathematical operations.

This also solves the problem pointed by Parr, that of the thoughts about Aleph: size of infinity, types of infinity, and etc. Now, one immediately understands how large the axis is when compared to its pieces, what is simply logical and makes it all coherent (small pieces are infinitum, larger pieces are infinita, and the whole lot is the interval between the infimum of the negative infinita and the supremum of the positive ones).

With this, 0.5 in the axis, where 1 is the size of the infinitum, means 0.5 infinitum, what immediately makes us associate 0.5 with uncountably many units contained in it, what is definitely different from considering half a unit only (like half a chocolate?).

We actually treat all the little bits of it in the same way that we treat the cells in a human body: The human body is just one but, for a person to claim to be a biologist, they must see it as a set of cells, uncountably many (so far).

From now onwards, we will write x infinitum or x infinita to refer to the amount x in real numbers and this way of writing will be acceptable also in the Universe of Physics, with us managing to still keep in mind the difference between what belongs to Mathematics and what belongs to Physics when using the proposed terminology, what is trivially a necessity.

What follows is that it will be square infinita meters, for instance, while things are in the context of Mathematics, instead of square meters. This dissociates Mathematics from reality of things, making it all more sensible, for the World of Mathematics will never fit reality with 100% accuracy: It will fit this way only the reality of the world of an abstract ruler (for the concrete ruler usually bears mistakes of at least physical order) or the world of a mathematical object. This is a necessary dissociation if we want to reduce the amount of vain discussions, unsolvable problems, and paradoxes in Science (those will be stopped by the time of generation now, as a consequence (at least those relating to the issues that we deal with here)).

We also propose that other elements, besides axe and title, become an essential part of the mathematical graphs. These elements form a quadruplet ((size of infinitum with scales), uniform numbers representation[1], origin, and time[2]), and they should always appear in the just-mentioned order, so that anyone reading a graph will know what the graph is about from seeing its heading. 'Time' should be the time when the data has been collected.

Notice that, this far in time, we do not have rules for the Cartesian Plane in what regards plotting elements (from the domain or from the image) of a function on it. Yet, The Plane is one of the most important mathematical entities ever created and everything else in Mathematics is found completely rules-tied.

We are now axiomatizing the Cartesian Plane (whatever is sensible), and this is a necessity, since every mathematical object, if truly mathematical, must be 'born' of some sort of axiom of definition, and this is the minimum requirement for it to 'belong' to Mathematics.

It is then worth mentioning that our infinitum should be logically founded. As an example, we may get a graph where every domain element has a distance of 0.5 from the previous domain element and the counter-domain elements vary by 1 and start on 0. We think that we are ready to draw this graph by 2 pm of the 9th of April of 2009. In this case, we may choose our infinitum to be 0.5 (smallest necessary 'slice' of the axis). Because the maximum detail of the data is one decimal place, we'd better choose this format for every spelled member of the axis. It looks wise to place the origin on zero, even though we would need all the details about the data to make the best logical decision. Our graph, in principle, then, should appear accompanied by the quadruplet ((0.5; 1), 10^{-1}, (0;0), 09/04/2009 at 2 pm) and its title.

The difference between the Infinitum World and the Cartesian Plane might be subtle, but it helps everyone involved, on a subconscious/psycholinguistic level, since, for instance, all are kept consciously aware of the presence of an infinite number of members in each non-degenerated interval of the real line. Besides, with our quadruplets, there is also conscious awareness of the difference between the infinitum of 0.5 and the infinitum of 1.3, for instance, that is, all understand immediately that there are more elements in one than in the other.

Some changes will not be a big deal. For instance, all operations that used to return infinity in Mathematics this far in time will return one of the options below instead:

1 To be used only in case we have decimals

2 Time is the 'time of creation' and the time of creation we refer to here is mentioned in more detail in another work of ours, which is currently in the shape of preprint. Please consult it to understand what we say here better

a) Sup Infta;
b) Inf -Infta.

All the operations that used to appear associated with 'infinity' will now appear associated, instead, with one of the elements above (a or b).

Originally, the symbol used for infinity was replacing a determined numeral in the sequence of the Roman numerals (see, for instance, [Weisstein 1999]). Notwithstanding, in Mathematics, we cannot accept symbols, or words, for its lingo that have already been used inside of it to mean something else, for every word in its lingo should be univocal. Therefore, we are fixing more historical mistakes than the ones we have initially waved with by producing this new notation, or set of new sigmatoids, for the mathematical lingo.

Worth remembering, as well, that the way in which infinity is now found in Mathematics makes us think that infinity is an actual 'number', which may appear in the mathematical operations (division, multiplication, and etc.). Notwithstanding, this is also incompatible with the current meaning of the entity created by John Wallis in 1655, that is, with the pair (sigmatoid; reference) that is currently associated with it.

Notice that, with our notation, we now have what is, in Mathematics, passive of inclusion in a division, in a multiplication, or in any other operation, what was not happening before, with the notation introduced by John Wallis. Even the immediate understanding of what is going on with the operation of division between *1* and *x*, for instance, using the example from [Weisstein 1999], when *x* goes to what should be the largest real number available, is now incredibly improved, for it is consistent and coherent with all pre-existent mathematical concepts that may be involved somehow in the operation 'limit'.

Up to now, a teacher would have to 'teach' infinity soon after introducing the concept 'limit', but, from now onwards, they will simply add the concept 'limit' instead, after all other mathematical concepts, so that it will not look like something 'forced', or 'manufactured', to almost clearly 'illegally' replace what should not be there.

4. Practice: One example

We now present a situation in which, previously, we would be using the Cartesian Plane as background. Now, we will be using the World of Infinitum instead. As we go through the example, we also write about some of the other advantages of the new system introduced by us here.

From [Dendane 2008]:

Problem 5:

The cost of producing x tools by a company is given by

C(x) = 1200 x + 5500 (in $)

a) What is the cost of 100 tools?
b) What is the cost of 101 tools?
c) Find the difference between the cost of 101 and 100 tools.
d) Find the slope of the graph of C.
e) Interpret the slope.

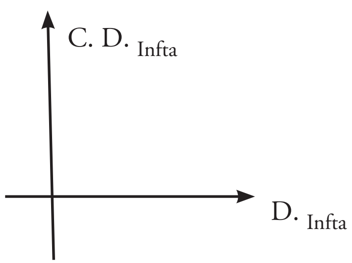
All the above questions are supposed to be answered without drawing a graph, trivially. However, in wishing to draw a graph, the best thing to do would be dividing this function by 1000. In this case, each unit of the previous function would be corresponding to one thousandth of itself. We would then have more than one scale to worry about, since we also have to adapt all to the size of the Infinitum.

Since the main intents of the problem are studying the 'slope' of the graph of the function, we can choose any domain number to work with. We choose $x = 0$ and $x = 1$ (we only need two for this exercise, trivially), attaining $C_{1}(0)=5.5$ and $C_{1}(1)=6.7$. With no need, we form one more ordered pair: *(2; 7.9)*.

This way, our Infinitum could be *1* for our horizontal axis and *1.2* for our vertical axis. The time of collection of the data may be assumed to be now, for it is not mentioned in the problem and we cannot find out the origin of the data (what data? What we have created?). The origin of the system may be the usual one: *(0;0)*. We would then end up with the following structure:

World of Infinitum (or WI)

((*1 tool*;*1.2 dollars (original cost ÷ 1000)*), 10^{-1},(*0*;*0*), *17/07/2009 at 7:02 pm*)



Thus, in the World of Infinitum, we would call our Cartesian-like axes Domain of Infinita (D. $_{Infta}$) and Counter-Domain of Infinita (C. D. $_{Infta}$), instead of Domain and Counter-Domain, usually referred to as *X* and *Y* in the Cartesian Plane.

From our horizontal axis, we would read the numbers *0*, *1*, and *2* at least. From the vertical axis, we would read the numbers *5.5*, *6.7*, and *7.9*. Even though we think of distance one for the vertical axis, it is still the usual ruler that commands physical position on the axis, so that the numbers will still have the equivalent to *1.2* cm of separation between them and the horizontal markers will have *1* cm between them. This is so that the graph, that is, the relationship between the variables involved, be preserved ($C(x)/1000 = 1.2\,x + 5.5$).

The enormous differences between the concrete world and the abstract one, and the need of a complex interface of conversion, or of communication, between them, are now perceived at a conscious level, making it easier, for the researchers, to understand in full all complexities involved, so that their insights may now quickly 'tune in' the right waves of 'possible emanation of inspiration'.

In reading from the graph, the importance of the information in between brackets is fundamental: Suppose that we read *5.5* from our vertical axis in WI. We then must perform any operations mentioned there to work out the 'impact', in the actual world, of our 'abstract graph'. In our case: $5.5 \times 1000 = 5{,}500$ dollars.

5. Conclusion

In this paper, we have introduced the World of Infinitum, a mathematical tool, as a needed replacement for the Cartesian Plane.

The World of Infinitum (WI) represents fantastic progress when compared to the Cartesian Plane, since it is a far more accurate description of all the entities involved.

Several inaccuracies in the theories deriving from 'graphical reasoning' seem to have been eliminated with this substitution.

In this paper, we also propose that we abandon both the symbol and the notion of 'infinity', as we know them this far in Mathematics, since we have proved that they constitute mistaken conceptions when the foundations of Mathematics are considered. To replace the concept and its symbols in what is sensible, we have created the concept of 'infinitum' and we have made it be represented, in its different mathematical functions, by an abbreviation and existent mathematical symbols.

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I. M. R. Pinheiro
P.O. Box 12396
A'Beckett St, Melbourne, Victoria, 8006.
Australia
E-mail: mrpprofessional@yahoo.com.